Midterm
Classical Mechanics

B. Math., 2nd Year, July - November 2024,
Date: September 12, 2024 (Wednesday)
Total points: 60.
Total time: 120 minutes

Provide arguments for steps which are not obvious. Any result written down without explanations will receive zero credit.

1. A uniform hoop of mass m and radius r is on top of a stationary cylinder of radius $R(> R)$. The only external force is that of gravity. The smaller hoop starts rolling without slipping from rest at the top of the bigger cylinder. At which point does the hoop roll off the cylinder?

   10 points

2. Obtain the equation of motion for a particle falling under gravity in the presence of a dissipational force of $\frac{1}{2}kv^2$. Find the velocity as a function of time and show that the maximum possible velocity is mg/k.

   10 points

3. A point particle moves under the influence of a force derivable from a generalized potential of the form

   $$U(\vec{r}, \vec{v}) = V(r) + \vec{\sigma}.\vec{L}$$

   where $\vec{r}$ is the position vector from the origin and $\vec{L}$ is the angular momentum about the origin. $\vec{\sigma}$ is a constant vector in space.

   (a) Find the components of the force acting on the particle in both Cartesian and spherical polar co-ordinates.

   (b) Find the equations of motion in spherical co-ordinates.

   15 points

4. Consider a particle moving under a central conservative force with the radial dependence of the potential $V(r) = -\frac{\alpha}{r^3}$. In a E-r diagram, for a fixed value of l, discuss the various possibilities of motion for different

energies. Pay particular attention to any case (if any) where the particle motion depends upon the initial position also, instead of only E.
15 points